ECONOMICS

Working Paper Series

No. 02-2021

Revealed statistical consumer theory

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Abstract: We provide a microfoundation for using aggregated data (e.g. mean purchases) when evaluating consumer choice data. We present a model of statistical consumer theory where the individual maximizes their utility with respect to a distribution of bundles that is constrained by a statistic of the distribution (e.g. mean expenditure). We show that this behavior is observationally equivalent to an individual whose preferences depend only on the statistic of the distribution. This means that despite working with distributions, the empirical content of the model only depends on a e 0.1912.2-0.029 o Tw (c))Tjrj -0.025 Tw ( )

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February, 2021

Abstract

We provide a microfoundation for using aggregated data (e.g. mean purchases) when evaluating consumer choice data. We present a model of statistical consumer theory where the individual maximizes their utility with respect to a distribution of bundles that is constrained by a statistic of the distribution (e.g. mean expenditure). We show that this behavior is observationally equivalent to an individual whose preferences depend only on the statistic of the distribution. This means that despite working with distributions, the empirical content of the model only depends on a nite-dimensional statistic. Statistical consumer theory neither nests

1 Introduction

Outside of experimental settings, data typically do not match the classic consumer problem of utility maximization subject to a budget constraint. A leading example is scanner data. Such data is highly disaggregated, typically at the transaction-level. This raises several concerns when attempting to apply consumer theory. First, it is unclear whether the budget of the consumer is at the transaction or for a longer period of time. A second concern is the \zeros" problem: a typical transaction involves zero quantities of many goods, though when examining many transactions an individual may purchase a wide variety of goods. In practice, researchers typically aggregate (sum up) the disaggregated data across time to address the zeros problem. In this case, budgetary constraints are in terms of aggregated data. For example, Echenique et al. (2011) process transaction-level data to form quantities for four-week periods and analyze this aggregated data without referencing the original primitive data.[1] Several questions arise from this procedure: Precisely, how does the budget enter? What information is lost when working directly with the aggregated data rather than transaction-level quantities?[2]

This paper addresses these questions by providing a demand theory for distributions.

over nitely many alternatives. Following Machina (1985), we take a \demand approach," but study the classical consumer setting with distributions over consumption bundles. We di er from Machina (1985) since distributions over bundles are an in nite dimensional object and we leverage price variation from the standard consumer problem. The main characterization with a mean expenditure constraint demonstrates an important dimension reduction aspect where an in nite dimensional model is observationally equivalent to a nite dimensional one.

Our approach di ers from random utility models (RUMs), which are the main paradigm to model stochastic choice and have a rich intellectual history following Thurstone (1927), Luce (1959), Block and Marschak (1960), Falmagne (1978), McFadden and Richter (1990), McFadden (2005), Kitamura and Stoye (2018 with budget constraints on average expenditure and relates this behavior to choice with selection of a particular bundle in $\mathbb{R}^L_+$ according to this distribution.

The primitive dataset $D :=$ $(p^t; m^t; ^t) : t \; 2 \; T$ consists of a nite number of triplets

of prices $p^t \; 2 \; \mathbb{R}^L_{++}$,,,, n(i)1(o)1t9552 Tfr267 7.548 0 Td 2.955 Td[160 Td 2.955 Td[160 Td 2.955 Td

Figure 1a, all bundles chosen with positive probability have expenditure equal to the average expenditure. In contrast, Figure 1b has bundles chosen with positive probability that are below (blue) or above (red) the average expenditure constraint. One could interpret purchases below average expenditure as \saving" behavior whereas a realization of purchases above average expenditure could be interpreted as \splurging" behavior. Moreover, this distribution has several realizations with zero purchases of di erent goods that are common in data. The distinction between saving and splurging purchases could

Clearly, with no additional restriction on the utility function, any dataset can be rationalized with a constant function U. For this reason, we restrict our attention to the class of locally nonsatiated

Mean acyclicity requires further comment. In particular, the condition applies to one element cycles $C = \{(t, t)\}$. It follows that maximization of a locally nonsatiated utility function requires that $\int (p^t \cdot x)\, d\mu^t(x) = m^t$, for all $t \in T$. Thus, the budget constraint must be binding for every observed choice. An important practical implication of this fact is that it is not crucial for the analyst to observe the expenditure level m^t. Thus, one only needs a dataset $(p^t; \mu^t)_{t \in T}$ while setting the expenditure level $m^t = \int (p^t \cdot x)\, d\mu^t(x)$ to check mean acyclicity.

The definition of mean acyclicity is equivalent to restrictions on the revealed preference relations. In particular, mean acyclicity coincides with the *generalized axiom of revealed preference* (GARP) on the revealed preference relation R so that

$$\mu^t R \mu^s \text{ implies not } \mu^s P \mu^t. \tag{4}$$

Mean acyclicity is a straightforward extension of GARP (as in Afriat, 1967; Diewert, 1973; Varian, 1982) to choices over probability measures, rather than consumption bundles. In fact, if the consumer chooses only degenerate lotteries, then GARP coincides with mean acyclicity. To see this, for all $t \in T$ a degenerate lottery satisfies $\mu^t = \delta_{x^t}$, where the latter denotes the Dirac measure concentrated at some $x^t \in \mathbb{R}^L_+$. Therefore, we have $\int (p^t \cdot x)\, d\mu^s(x) = p^t \cdot x^s$ for all $t, s \in T$, which reduces mean acyclicity to GARP.

Finally, by Lemma 11.45 in Aliprantis and Border (2006), we have

$$\int (p \cdot x)\, d\mu(x) = p \cdot \int x\, d\mu(x)$$

for all $\mu \in \Delta$ and $t \in T$. This implies that all relevant information for mean acyclicity is summarized by the L-dimensional mean bundle $\int x\, d\mu(x)$ for the distribution μ. In practice, estimating mean bundles rather than the whole distribution μ^t is sufficient to check the mean acyclicity condition.

Since we can represent mean acyclicity on mean bundles, it is natural to study utility

If the dataset can be rationalized with a mean choice model, then any utility function that rationalizes the data only depends on the mean bundle (a vector), rather than all information in the distribution. In the main theorem below, we show that locally nonsatiated preferences are observationally equivalent to the mean choice model when the sets of distributions are restricted by the average expenditure budget de ned in equation (1).

Theorem 1. For any set of observations D^A

^{t} only matters insofar as it leads to error in estimating the mean of consumption. This is because checking mean acyclicity is possible by checking restrictions on means. We leverage the dimension reduction aspect to incorporate variability in empirically relevant datasets in Allen et al. (2021).

2.3 Relation to a random income model

The model of distributional preferences above may seem stylized, but we show it generalizes a class of random utility models where income is also random and unobserved. In applied analysis, it is often assumed that the income of an individual is equal to expenditure on goods. However, since income is often unobserved, it might make more sense to treat it as a random variable. If preferences and income are random, then we are essentially in the case of the Sonnenschein-Mantel-Debreu (Debreu, 1974) anything goes result for mean demands[12]. However, we show that a random quasilinear utility maximizer with random income is rationalizable with a mean choice model.

To formalize this, let (; ") be random variables that govern random utility functions and income. We assume these variables are independent of prices, but allow preferences to be potentially correlated with income. We give a precise de nition of this model below.

De nition 3. A random quasilinear utility and income model with random variables (; ") has individuals make choices according to

$$\max_{(x;y)2R^L_+ \quad R} \quad u(x; \ ) + y$$

$$\text{s.t.} \quad p \ \ x + y \quad m("):$$

Thus, the realization of the random variable gives a random draw of preferences, while the random variable " governs the realization of income. To show the relation to a mean choice model and distributional choice, we will make some assumptions. For technical simplicity, we suppose that (; ") takes values in the nite set N E and for each realization of the utility function $u : R^L_+ \quad N \ ! \quad R$ yields a unique maximizer for all prices f p^t g$_{t2T}$. We let $x^{;t}$(; ") denote the unique choice given price p^t and unobservables (; "). To map to our previous analysis, given a price p^t, a distribution of choices arises

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because $x^{*,t}(\cdot,\varepsilon)$ is random due to $(\cdot,\varepsilon)$. The quasilinear model has recently been studied in Brown and Calsamiglia (2007) and Allen and Rehbeck (2020a,b).

The paper by Allen and Rehbeck (2020b) shows that the solutions to the maximization problem satisfy cyclic monotonicity (cf. Rockafellar, 1970) on the expectation of the choices so that for any sequence $\{t_m\}_{m=1}^{M}$ with is rationalized by a random utility model (RUM) if there is a probability measure μ over the space of functions $\mathcal{U}$ such that, for all $t \in T$:

$$\pi^t(O) = \mu\left(u \in \mathcal{U} : \operatorname{argmax}_{y \in B^t} u(x) \in O\right); \tag{6}$$

for any measurable subset $O \subseteq \mathbb{R}^L_+$, where the argmax set is a singleton since $\mathcal{U}$ consists of strictly quasiconcave functions. In other words, the probability of choosing a bundle in the set O is equal to the probability of drawing a utility function that is maximized over B^t at some point in the set O. For a linear programming characterization of RUM see McFadden and Richter (1990), McFadden (2005), and Kitamura and Stoye (2018).[13]

Distributions of choices generated by a random utility model are in the set

$$C(p;m) := \left\{\pi \in \Delta : \pi(B(p;m)) = 1\right\}.$$

A key difference between the mean choice model and random utility model is that budget sets $A(p;m)$ and $C(p;m)$ are different. The mean choice model allows choice of consumption bundles that exceed the expenditure level m^t, which is not allowed in random utility models. Recall that as emphasized above, for the mean choice model only the average expenditure need be measured. The random utility model has a fixed budget, in which case the average expenditure is the same as the expenditure for each realization of the random utility. To further compare the models, we restrict attention to distributions when the support of π^t is a subset of B^t,[14] for all $t \in T$. We show that mean choice models neither nest nor are nested in random utility models for such distributions.

In Example 1, we discuss a dataset that can be rationalized only by a mean choice model. Here, there is no RUM that can generate the observations. Despite this, the mean behavior is consistent with mean acyclicity. In contrast, the dataset in Example 2 is only rationalizable by a RUM. Since the models describe different behavior, one can discriminate between mean choice models and RUMs using field data or experiments.

Example 1. Let a primitive dataset be given by $D = \left\{(p^1; m^1; \pi^1); (p^2; m^2; \pi^2)\right\}$, where $p^1 = (2; 1)$, $p^2 = (1; 2)$, and $m^1 = m^2 = 1$. In addition, suppose that measure π^1 assigns probability $7/12$ to bundle $(1/2; 0)$ and $5/12$ to $(0; 1)$, while π^2 assigns probability $7/12$ to $(0; 1/2)$ and $5/12$ to $(1; 0)$.

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[13] Alternatively, random utility models are characterized by the axiom of revealed stochastic preferences in McFadden and Richter (1990) and McFadden (2005) which is conceptually more similar to GARP.

[14] The support of π^t is the smallest closed set K such that $\pi^t(K) = 1$.

Both $\int (p^1 \cdot x)\,d\mu^2(x)$ and $\int (p^2 \cdot x)\,d\mu^1(x)$ are equal to $13/12 > 1 = m^1 = m^2$, which suffices for the set of observations to satisfy mean acyclicity and, thus, be rationalizable by a mean choice model. Equivalently, the means of distributions μ^1, μ^2 are given by $\bar{x}_{\mu^1} = (7/24; 5/12)$, $\bar{x}_{\mu^2} = (5/12; 7/24)$, respectively, where $p_1 \cdot \bar{x}_{\mu^2} = p_2 \cdot \bar{x}_{\mu^1} = 13/12 > 1$. See Figure 2 for a graphical interpretation.

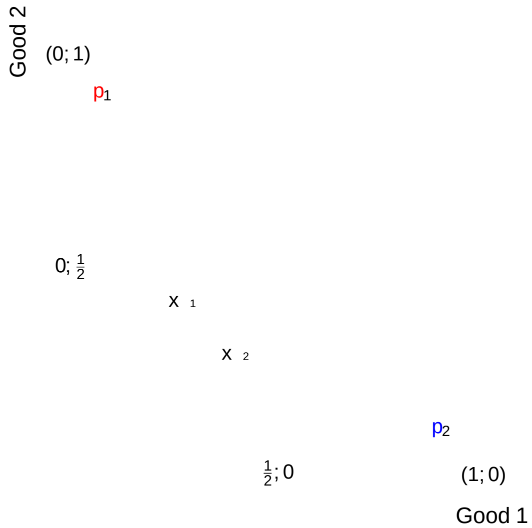

Figure 2: Graphical interpretation of the dataset in Example 1.

In contrast, the data are inconsistent with the random utility model. Indeed, since $p^1 \cdot (0; 1/2) = 1/2 < m^1$, there must be probability of at least $7/12$ on utilities where bundle $(0; 1/2)$ is strictly inferior to $(1/2; 0)$. Analogously, as $p^2 \cdot (1/2; 0) = 1/2 < m^2$, at least a probability of $7/12$ on utilities must rank $(0; 1/2)$ strictly over $(1/2; 0)$. However, this would imply that for a probability of at least $1/6$ of all utilities we would have both $u(1/2; 0) > u(0; 1/2)$ and $u(1/2; 0) < u(0; 1/2)$, which yields a contradiction.

Example 2. Let the primitive dataset be given by $D = \{(p^1; m^1; \mu^1); (p^2; m^2; \mu^2)\}$ where $p^1 = (2; 1)$, $p^2 = (1; 2)$, and $m^1 = m^2 = 1$; moreover, the measure μ^1 assigns probability $1/2$ to bundles $(1/2; 0)$ and $(1/4; 1/2)$, while μ^2 assigns probability $1/2$ to $(0; 1/2)$ and $(1/2; 1/4)$. See Figure 3 for a graphical representation.

One can easily show that the dataset violates mean acyclicity. At the same time, it is straightforward to show that the set of observations can be rationalized with a random utility model. Clearly, one can always find a function $u_1 : \mathbb{R}^2_+ \to \mathbb{R}$ in $\mathcal{U}$ that is uniquely maximized at $(1/2; 0)$ over $B^1 := \{x \in \mathbb{R}^2_+ : p^1 \cdot x \leq 1\}$ and uniquely maximized at

$(1=2; 1=4)$ over $B^2 :=$ $x \ 2 \ R^2$

is feasible if $g^t \ S(\ ) \quad 0$. Here a general dataset is given by $D^G = \ (g^t;\ {}^t) : t \ 2 \ T$.
Here we make the high level assumption that

Theorem 2. For any set of observations D^G

where $E(\ ) = \quad E_{\ell}(\ )\ _{\ell=1}^{L}$ and $Var(\ ) = \quad Var_{\ell}(\ )\ _{\ell=1}^{L}$.

This shows that it is observationally equivalent to behavior of preferences that depend only on the mean and variance of the distribution of choices. We note that the preferences rationalizes the data where $L = \{1, \ldots, L\}$ and $J = \{1, \ldots, J\}$. Thus, one can rationalize the choice of distributional choice that only depend on the moments of the distribution. Straightforward extensions of this could place restrictions on moments across goods. As in the previous example, one could also add the restriction of average expenditure constraints to the function that described the budget constraint and still obtain a preference that only depends on moments of the distribution.

We note that as additional moments of a distribution are modeled in the constraints, there are fewer revealed preference comparisons. To see this, note that if one adds an additional moment restriction to the constraints, then a distribution must satisfy an additional inequality to be revealed preferred. Thus, introducing additional moments to the constraints will necessarily describe more datasets. One potentially interesting exercise would be to look for the least moment restrictions that rationalize a dataset.

5 Conclusion

This is the first paper to provide a microfoundation for using aggregated data to examine consumer preferences. Even though many papers empirically analyze models using aggregate choices, until this paper there was no formal microfoundation that justified this practice. We show that if individuals have a preference for randomization, then it is without loss of generality to use data on aggregate choices.

More broadly, this paper relates to the growing literature on stochastic choice. For example, we show how a random quasilinear utility model with random incomes is nested in the approach. We also show that in practice statistical choice models can be differentiated from random utility models that have a fixed budget. While the main results study the average expenditure constraint, we show that the results also apply to general constraints that depend on a statistic of the distribution. We show how this can be used in practice to characterize a generalization of mean-variance preferences and preferences that depend on arbitrary moments.

One implication of the results for the mean choice model is that welfare analysis is possible by building on existing results from the standard consumer problem. Moreover, since it is without loss of generality to study models that depend on mean consumption, an applied researcher can use their favorite functional forms from consumer theory. For example, one can use a Cobb-Douglas model and replace the choice of consumption bundles with means. Lastly, in ongoing work (Allen et al., 2021) we present a statistical have $g^t(\ \ ^s) = 0$, for all $(t; s)$ 2 C. By Lemma 2 in Forges and Minelli (2009), there are numbers$f$ $\ ^t g_{t 2 T}$ and strictly positive numbers$f$ $\ ^t g_{t 2 T}$ such that $\ ^s \quad \ ^t + \ ^t g^t \ S(\ \ ^s)$, for all $t; s$ 2 T

fact that f is concave and strictly increasing follows directly from the construction of the function in the proof of Theorem 2 and the fact that for all $t \in T$ the function g^t is concave (in fact, linear) and strictly increasing. □

Proof of Proposition 1. Suppose that the data is generated by a random quasilinear utility and income model as in the statement of the proposition. For each p^t, let

$$\left(x^{*,t}(\theta,\varepsilon), y^{*,t}(\theta,\varepsilon)\right) := \operatorname*{argmax}_{(x,y)\in\mathbb{R}^L_+\times\mathbb{R}} \left\{ U(x,\theta) + y : p^t \cdot x + y \leq m(\varepsilon) \right\}$$

be the maximizer of choices when the values (θ,ε) are realized by the random variables. Here $y^{*,t}(\theta,\varepsilon) = m(\varepsilon) - p^t \cdot x^{*,t}(\theta,\varepsilon)$.

Conditioning on the realization of the random variables, we can compare this to the $x^{*,s}(\theta,\varepsilon)$ when purchased at prices p^t. It follows that

$$U\left(x^{*,t}(\theta,\varepsilon),\theta\right) - p^t x^{*,t}(\theta,\varepsilon) \geq U\left(x^{*,s}(\theta,\varepsilon),\theta\right) - p^t x^{*,s}(\theta,\varepsilon)$$

where finiteness of the utility numbers is ensured by the existence of maximizers. Still conditioning on (θ,ε), we can look at any sequence $\{t_m\}_{m=1}^M$ with $t_m \in T$ and get that

$$\sum_{m=1}^M p^{t_m} \cdot \left(x^{*,t_m}(\theta,\varepsilon) - x^{*,t_{m+1}}(\theta,\varepsilon)\right) \leq 0$$

where $t_{M+1} = t_1$. Taking expectations over (θ,ε), it follows that

$$\sum_{m=1}^M p^{t_m} \cdot \left(\mathbb{E}\, x^{*,t_m}(\theta,\varepsilon) - \mathbb{E}\, x^{*,t_{m+1}}(\theta,\varepsilon)\right) \leq 0$$

where $t_{M+1} = t_1$ by linearity of expectations. This holds for any dataset generated by a random quasilinear utility and income maximizer.

To see that mean acyclicity is satisfied, suppose by contradiction that there is a cycle. It follows that there exists a sequence $\{\bar{t}_m\}_{m=1}^M$ with $\bar{t}_m \in T$ where

$$p^{\bar{t}_m} \cdot \mathbb{E}\, x^{*,\bar{t}_{m+1}}(\theta,\varepsilon) \leq p^{\bar{t}_m} \cdot \mathbb{E}\, x^{*,\bar{t}_m}(\theta,\varepsilon)$$

with at least one inequality strict where $\bar{t}_{M+1} = \bar{t}_1$. Summing these inequalities up yields

$$\sum_{m=1}^M p^{t_m} \cdot \left(\mathbb{E}\, x^{*,t_m}(\theta,\varepsilon) - \mathbb{E}\, x^{*,t_{m+1}}(\theta,\varepsilon)\right) > 0$$

which contradicts that the data is generated by a random quasilinear utility. □

Appendix B First order stochastic dominance

Here we discuss properties of the rst order stochastic dominance. Let $_X$ denote a Borel space of probability distributions over some X R^L. We consider the usual partial order over R^L, i.e., for $x; y$ 2 X R^L, x y if and only if x_i y_i for each ` = 1; : : : ; L. The distribution rst order stochastically dominates , or , whenever $\int f(x)d(x)$ $\int f(x)d(x)$, for any measurable, bounded, and nondecreasing function $f : X !$ R.

One can show that is a partial order over $_X$. This follows from Theorem 2 in Kamae and Krengel (1978) and the fact that R^L is a Polish space.

Lemma B.1. Suppose that , for some ; 2 $_X$. There is a probability space (; F ;) and random variables X ; X : ! X such that

(i) X and X are distributed according to and respectively, i.e., for any Borel measurable set O X we have

$$(O) = \quad ! \ 2 \ : X (!) \ 2 \ O \quad \text{and} \quad (O) = \quad ! \ 2 \ : X (!) \ 2 \ O \ ;$$

(ii) X (!) X (!), for all ! 2 .

See Lemma 4 in Kamae and Krengel (1978) for the proof. We say that strictly dominates , and denote it by , if and 6= . Using Lemma B.1, it is easy to show that we have if and only if there are random variables X ; X : ! X such that X (!) X (!), for all ! 2 , where the inequality is strict for all ! in some measurable set F such that (F) > 0. We now prove a series of lemmas.

Lemma B.2. The distribution rst order stochastically dominates , or , if and only if (D) (D), for any measurable and upward comprehensive set D.[20]

Proof. We prove the implication ()) by contradiction. Suppose that , but there is some measurable, upward comprehensive set D such that (D) < (D). Let $_D$ be the indicator function, taking values $_D(x) = 0$, for x 2= D, and $_D(x) = 1$ otherwise. The is increasing. Since the simple function is defined on a measurable set, it is measurable. However, it must be that $\int \mathbb{1}_D(x)d\ (x) = \ (D) < \ (D) = \int \mathbb{1}_D(x)d\ (x)$, which contradicts that first order stochastic dominates .

The converse follows directly from the definition of Lebesgue integration. Suppose that, for any upward comprehensive and measurable set D, we have $(D) \quad (D)$. Clearly, D is upward comprehensive if and only if its complement $\mathbb{R}^L \setminus D$ is downward comprehensive. Thus, for any such set E, we have $(E) \quad (E)$.

Take any bounded, measurable, and increasing function $f : \mathbb{R}^L \to \mathbb{R}$. Clearly, for all $r \in \mathbb{R}$ any sets of the form $\{y \in \mathbb{R}^L : f(y) > r\}$ and $\{y \in \mathbb{R}^L : f(y) < r\}$ are upward and downward comprehensive, respectively. Moreover, they are both measurable, by measurability of f. This implies that

$$
\begin{aligned}
\int f(x)d\ (x) &= \int_0^1 \ \{x \in \mathbb{R}^L : f(x) > y\}\, dy \quad \int_{-1}^0 \ \{x \in \mathbb{R}^L : f(x) < y\}\, dy \\
& \quad \int_0^1 \ \{x \in \mathbb{R}^L : f(x) > y\}\, dy \quad \int_{-1}^0 \ \{x \in \mathbb{R}^L : f(x) < y\}\, dy \\
&= \int f(x)d\ (x).
\end{aligned}
$$

Since this is true for any increasing function f, the proof is complete. □

Before we state the next result, a function $f : X \to \mathbb{R}$ is strictly increasing if $x'_\ell \quad x_\ell$, for all $\ell = 1, \ldots, L$, and $x'_\ell > x_\ell$, for some ℓ, implies $f(x') > f(x)$, for any $x, x' \in X$.

Lemma B.3. Suppose that , for some $\ , \ \in \ _X$. For any strictly increasing function $f : X \to \mathbb{R}$, we have $\int f(x)d\ (x) > \int f(x)d\ (x)$.

Proof. Given that , Lemma B.1 implies that there is a probability space $(\ ; F; \ )$ and random variables $X\ , X\ : \ \to X$ that are distributed according to , respectively, and $X\ (!) \quad X\ (!)$, for all $! \in$. Since , let $'$ be defined so that

$$
' = \{ ! \in \ : X\ (!) > X\ (!) \}:
$$

where $(\ ') > 0$ (recall Lemma B.1). For any strictly increasing $f : X \to \mathbb{R}$, we have

$$
\begin{aligned}
\int f(x)d\ (x) \quad \int f(x)d\ (x) &= \int \left[f\big(X\ (!)\big) \quad f\big(X\ (!)\big) \right] d\ (!) \\
&= \int_{\ '} \left[f\big(X\ (!)\big) \quad f\big(X\ (!)\big) \right] d\ (!) > 0.
\end{aligned}
$$

This completes the proof. □

Lemma B.4. Suppose that $X + R^L_+ \quad X$. For any measure $\quad 2 \quad _X$ and its neighborhood, we have $\quad$, for some $\quad$ in the neighborhood.

Proof. We show that for any $\quad 2 \quad$ there is a sequence $f \quad ^k g$ in $\quad$ that weakly converges to $\quad$ and $\quad ^k \quad$, for all k. Take any probability space $(\quad ; F ; \quad)$ and the random variable $X \quad : \quad ! \quad X$ that is distributed according to $\quad$, i.e., for any measurable $O \quad X$ we have

$$(O) = \quad ! \ 2 \quad : \ X \ (!) \ 2 \ O \quad :$$

Take any sequence $f X^k g$ of random variables $X^k : \quad ! \quad R$ that pointwise converge to X and satisfy $X^k(!) > X \quad (!)$, for all $! \ 2 \quad$.

For each k, de ne a probability measure $\quad ^k$ so that for any measurable $O \quad X$

$$^k(O) := \quad ! \ 2 \quad : \ X^k(!) \ 2 \ O \quad :$$

Since $X + R^L_+ \quad X$, we have $\quad ^k \ 2 \quad _X$. Moreover, for any measurable, upward comprehensive set D, it must be that

$$^k(D) = \quad ! \ 2 \quad : \ X^k(!) \ 2 \ D \qquad ! \ 2 \quad : \ X \ (!) \quad ! \qquad , \mathbf{D}$$

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